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AUTHOR Tatsuoka, Kikumi K.; Tatsuoka, Maurice M.
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ABSTRACT

The study examines the rule space model, a probabilistic model capable of measuring cognitive skill acquisition and of diagnosing erroneous rules of operation in a procedural domain. The model involves two important components: (1) determination of a set of bug distributions (bug density functions representing clusters around the rules); and (2) establishment of decision rules for classifying an observed response pattern into one of the clusters around the rules and computing error probabilities. This study further discusses the theoretical foundation of the model by introducing "bug distribution" and hypothesis testing (Bayes' decision rules for minimum errors). The model does not require a large-scale computation. It is helpful in areas of research and teaching. It can be used with microcomputers for testing hypotheses and validating data with probabilistically-sound information, and it can improve and modify information for the cluster ellipses as more students are added to research projects. (LMO)

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BUG DISTRIBUTION AND PATTERN CLASSIFICATION

KIKUMI K. TATSUOKA

MAURICE M. TATSUOKA

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A model (called rule space) which permits measuring cognitive skill acquisition, diagnosing cognitive errors, detecting the weaknesses and strengths of knowledge possessed by individuals was introduced earlier. This study further discusses the theoretical foundation of the model by introducing "Bug Distribution" and hypothesis testing (Bayes' decision rules for minimum errors) for classifying an individual into his/her most plausible latent state of knowledge. The model is illustrated with the domain of fraction arithmetic and compared with the results obtained from a conventional Artificial Intelligence approach.

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Copies of this report may be requested from Kikumi K. Tatsuoka, Computer-based Education Research Laboratory, University of Illinois, 103 S. Mathews St., Rm. 252, Urbana, IL 61801.

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Introduction

Several deterministic methods commonly used in Artifical Intelligence have been applied to develop problem-solving programs, or error-diagnostic systems. These methods have successfully diagnosed many erroneous rules of operation in arithmetic, algebra, and some science domains. The results of such error analyses have contributed to our current understanding of human thinking and reasoning.

These approaches, however, lack taking the variability of response errors into account, and they also depend on a specific model of problem solving. Therefore, they often cannot diagnose responses affected by random errors (sometimes called "slips") or produced by innovative thinking that is not taken into account by the current models. It is very difficult to develop a computer program whose underlying algorithms for solving a problem represents a wide range of individual differences. Yet, when these diagnostic systems are used in educational practice, they must be capable of evaluating any responses on test-items, inconsistent performances as well as those yielded by creative thinking. Therefore, we need a model that is capable of diagnosing non-systematic cognitive errors and is also capable of evaluating non-conventional problem-solving activities.

Tatsuoka and her associates (Tatsuoka, 1985, 1984a; Tatsuoka & Linn, 1983; Tatsuoka & Tatsuoka, 1983, 1982) have developed such a model called rule space and have successfully applied it to diagnose misconceptions possessed by students in signed-number and fraction arithmetic. The model maps all response patterns into a set of ordered pairs, the latent ability variable θ and one of the IRT based caution indices (ξ) introduced by Tatsuoka (1984a). However, the approach used in their model lacks, somehow, a sound statistical foundation in expressing

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random errors when a specific rule is applied for solving a problem. The simulation study by Tatsuoka and Baillie (1982) showed that the response patterns yielded by not-perfect-applications of a specific erroneous rule of operation in a procedural domain form a cluster around the rule. Moreover, they found empirically that the two random variables, θ and ζ obtained from those response patterns in the cluster follow a multivariate normal distribution. This cluster around a rule is called a "bug distribution" hereafter. The theoretical foundation of this empirical evidence will be discussed in this study. First, a brief description of the probabilistic model introduced in Tatsuoka (1985) will be given. Then the connection of each "bug distribution" to the model will be discussed in the conjunction with the theory of statistical pattern classification and recognition.

Distribution of Responses around an Erroneous Rule

The responses around a particular rule of operation in a procedural domain which are produced by not-perfectly-consistent applications of the rule to the test items form a cluster. They include responses which deviate, in various degrees of remoteness, from the response generated by the rule. When these discrepancies are observed, they are considered as response errors. These response errors are called "slips" by cognitive scientists (Brown & VanLehn, 1980). The properties of such responses around a given erroneous rule will be investigated in this section.

First, the probability of having a "slip" on item j (j=1,2,...,n) is assumed to be the same value, p for all items and it will be called "slip probability" in this paper. Let us denote an arbitrary rule for which the total score is r by Rule R and let the corresponding response pattern be:

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(1) $$R = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_r \\ x_{r+1} \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \; x_1 = x_2 \; \ldots = x_r = 1, \text{ and } x_{r+1} = \ldots = x_n = 0.$$

The response patterns existing one slip away from Rule R are of two kinds: a slip of "one to zero" occurring at $1 \leq j \leq r$ and "zero to one" at $r < j \leq n$. The number of response patterns having one slip is therefore $\binom{r}{1}\binom{n-r}{0} + \binom{r}{0}\binom{n-r}{1}$, and the probability of having one slip on items $j=1,\ldots,n$ is given by $\binom{r}{1} p^1 (1-p)^{r-1} \binom{n-r}{0} p^0 (1-p)^{n-r} +$ $\binom{r}{0} p^0 (1-p)^r \binom{n-r}{1} p^1 (1-p)^{n-r-1}$ if the probability p is the same for all items, $j=1,\ldots,n$. Therefore the following equation (2) is obtained;

(2) Prob (x_j - 1 for some $j=1,\ldots,r$ or x_j + 1 for some $j=r+1,\ldots,n$) = Prob (having a slip on an item) = $\{\binom{r}{1}\binom{n-r}{0} + \binom{r}{0}\binom{n-r}{1}\} p^1(1-p)^{n-1}$.

Similarly, the probability of having two slips on the items is given by Equation (3) as follows:

(3) Prob (having two slips on the items) = $\{\binom{r}{2}\binom{n-r}{0} + \binom{r}{1}\binom{n-r}{1} + \binom{r}{0}\binom{n-r}{2}\} p^2 (1-p)^{n-2}$.

In general, the probability of having k slips on the items is given by:

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(4) Prob (having k slips on the items)

$$= \left(\sum_{k_1+k_2=k} \binom{r}{k_1}\binom{n-r}{k_2} \right) p^k (1-p)^{n-k} .$$

The generating function of the distribution of frequencies up to k slips will be given by Equation (5) as follows:

$$(5) \quad \sum_{s \le k} \text{Prob (having s slips)} = \sum_{s \le k} \left\{ \sum_{s_1+s_2=s} \binom{r}{s_1}\binom{n-r}{s_2} \right\} p^s (1-p)^{n-s} .$$

Since the coefficient term inside the braces equals $\binom{n}{s}$, Equation (5) will be simply a binomial distribution, given by Equation (6).

$$(6) \quad \sum_{s \le k} \text{Prob (having s slips)} = \sum_{s \le k} \binom{n}{s} p^s (1-p)^{n-s} .$$

Therefore, a cluster around Rule R which consists of response patterns including various numbers of slips (not-perfectly-consistent application of Rule R) has a frequency distribution of a binomial form with the equal slip probability p for the items. One weakness in Equation (6) is that the value of p is not known and it is very unlikely that the value of p is constant over the test items. If we assume each item has an unique slip probability, then the binomial distribution expressed by Equation (6) will be a compound binomial distribution. Equation (7) is the generating function of the compound binomial distribution.

$$(7) \quad \sum_{s \le k} \text{prob (having s slips)} = \sum_{s \le k} \prod_{j}^{k} (p_j + q_j)$$

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Before an approximation of the slip probabilities p_j is discussed, the rule-space concept will be briefly introduced in the next section.

A Brief Summary of the Probabilistic model Rule Space

One of the purposes of the model, the rule space, is to interpret semantically the relationships among various erroneous rules and the right rule, and compare the characteristic of each rule to the right rule or other rules. An analogy for the underlying motivation of seeking a norm-referenced characteristic of "bug behavior" may be found in the theory and practice of norm-referenced tests. This starts by selecting the right rule as a norm and then comparing the other erroneous rules to the characteristic of the norm. By doing so, the psychometric behavior of "bugs" as compared with the right rule, understanding why and how various misconceptions are related and transformed from one to another will be explained more clearly than by just describing the list of bugs.

The rule space model begins by mapping all possible binary response patterns into a vector space of $((\theta, \zeta))$, where θ is the latent ability variable in Item Response Theory (IRT) and ζ (or $\zeta(\underset{\sim}{x};\theta)$) is one of the IRT based caution indices (Tatsuoka, 1984a; Tatsuoka & Linn, 1983). The mapping function $f(\underset{\sim}{x})$ is expressed as an inner product of two residual vectors, $\underset{\sim}{P}(\theta) - \underset{\sim}{x}$ and $\underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)$ where $P_j(\theta)$, $j=1,\ldots,n$ are the one- or two-parameter logistic-model probabilities, $\underset{\sim}{x}$ is a binary response vector and $T(\underset{\sim}{\theta})$ is the mean vector of the logistic probabilities. $f(\underset{\sim}{x})$ is a linear mapping function between $\underset{\sim}{x}$ and ζ at a given level of θ, and the response patterns having the same sufficient statistics for the maximum likelihood estimate $\hat{\theta}$ of θ are dispersed into different locations on the line of $\theta = \hat{\theta}$. For example, on a 100-item test, there are 4950 different response patterns

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having the total score of 2. The ζ's for the 4950 binary patterns will be distributed between ζ_{min} and ζ_{max}, where ζ_{min} is obtained from the pattern having 1 for the two easiest items and zeros elsewhere, and ζ_{max} is from the pattern having 1 for the two most difficult items. $f(\underset{\sim}{x})$ has the expectation zero and variance $\sum_{j=1}^{n} P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2$ (Tatsuoka, 1985). Since the expectation of the random variable x_j(j=1.,,,.n) is $P_j(\theta)$, the expectation of a vector $\underset{\sim}{x}$ is $\underset{\sim}{P}(\theta)$ whose jth component is $P_j(\theta)$. The vector $\underset{\sim}{P}(\theta)$ will be mapped to zero as shown in (8), thus the pattern corresponds to $(\theta,0)$ in the rule space.

(8) $f(\underset{\sim}{P}(\theta)) = 0$

As for an erroneous rule R, the response vector $\underset{\sim}{R}$ given by (1) will be mapped onto $(\theta_R, f(\underset{\sim}{R},\theta_R))$, where the ζ value is $\sum_{j=1}^{n}(P_j(\theta) - R_j)(P_j(\theta) - T(\theta))$, and is given by (9). That is,

(9) $f(\underset{\sim}{R}) = -\sum_{j=1}^{r} Q_j(\theta_R)(P_j(\theta_R) - T(\theta_R) + \sum_{j=r+1}^{n} P_j(\theta_R)(P_j(\theta_R) - T(\theta_R))$.

Similarly, all the response vectors resulting from several slips around rule R will be mapped into the vicinity of $(\theta_R, f(\underset{\sim}{R}))$ in the rule space and form a cluster (called the cluster around R hereafter). Figure 1 shows computer-simulated examples of such clusters done on the PLATO system.

Insert Figure 1 about here

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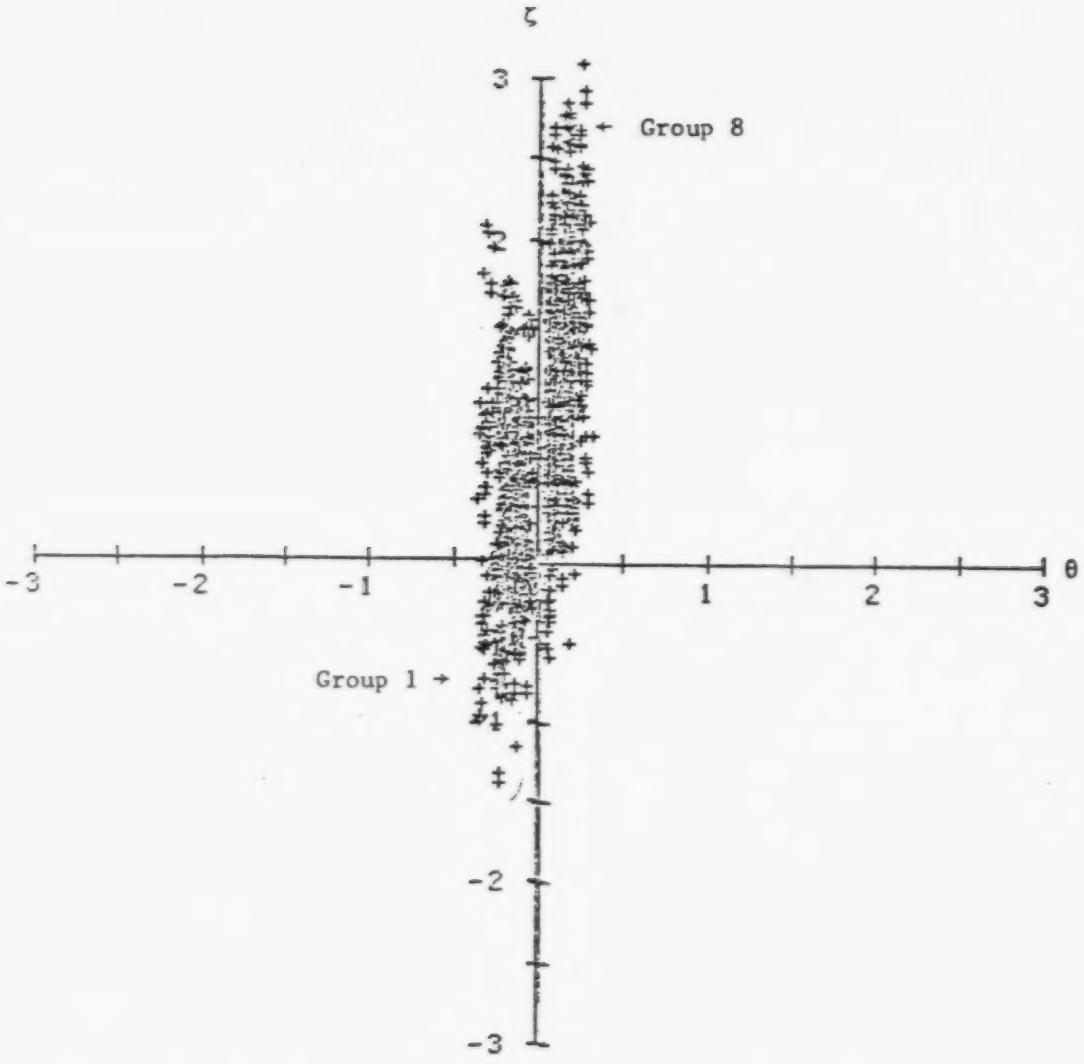

Figure 1: Two Clusters of Groups 1 and 8 with Two Slips. n=631

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The two variables θ and $f(\underset{\sim}{x})$ are mutually uncorrelated so their covariance matrix has a diagonal form as follows;

$$(10)\quad \begin{bmatrix} \mathrm{var}(\hat{\theta}) & 0 \\ 0 & \mathrm{var}(f(\underset{\sim}{x})) \end{bmatrix} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & \Sigma P_j(\hat{\theta})Q_j(\hat{\theta})(P_j(\hat{\theta}) - T(\hat{\theta}))^2 \end{bmatrix}$$

where $I(\theta)$ is the information function of the test and is given by $\Sigma a_j^2 P_j(\theta)Q_j(\theta)$ where the a_j $(j=1,\ldots,n)$ are item discriminating powers.

Let us map all response patterns of the test, including clusters around various rules into the Cartesian product space of $\hat{\theta}$ and $f(\underset{\sim}{x})$, where

$$(11)\quad f(\underset{\sim}{x}) = (\underset{\sim}{P}(\theta), \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)) - (\underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta))$$

$$\text{or} \qquad = K(\theta) - \sum_{j=1}^{n} x_j(P_j(\theta) - T(\theta)),$$

In particular, Rule R itself will be mapped as

$$\underset{\sim}{R} = \underset{\sim}{x} \rightarrow (\hat{\theta}_R, f(\underset{\sim}{R})),$$

where $f(\underset{\sim}{R})$ is given by Equation (9). The variance of the cluster around R will be expressed by using the slip probability of item j, p_j as follows;

$$(12)\quad \mathrm{Var}(\text{the cluster around R}) = \Sigma\, p_j q_j (P_j(\theta_R) - T(\theta_R))^2.$$

The quantities p_j and q_j are associated with Rule R as well as with item j, and their values are unknown. However, if the ordered pair (θ_R, ζ_R) in the rule space falls close to the θ axis, then p_j and q_j may be approximated by the logistic probability $P_j(\theta_R)$ and its complement $Q_j(\theta_R) = 1 - P_j(\theta_R)$,

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respectively, without too much loss of accuracy. If p_j and q_j are thus approximated, then the variance of Equation (12) will be the same as the variance of the mapping function $f(x)$; that is

$$(13) \quad \text{Var}(\zeta \text{ in the cluster around } R) \simeq \Sigma\, P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2$$

The variance of θ in any cluster, on the other hand, is given by the reciprocal $1/I(\theta)$ of the information function, which can be computed as

$$(14) \quad \text{Var}(\theta \text{ in the cluster around } R) = 1/I(\theta_R)$$
$$= 1/\Sigma\, a^2_j\, P_j(\theta_R)Q_j(\theta_R)$$
where $a_j = 1$ for the one-parameter logistic model.

The above two variances, along with the fact that ζ and $\hat{\theta}$ are uncorrelated, plus the reasonable assumption that they have a bivariate normal distribution, allow us to construct any desired percent ellipse around each rule point $\underset{\sim}{R}$. The upshot is that, if all erroneous rules (and the correct one) were to be mapped into the rule space along with their neighboring response patterns representing random slips from them, the resulting topography would be something like what is seen in Figure 2. That is, the population of points would exhibit modal densities at many rule points that each forms the center of an enveloping ellipse with the density of points getting rarer as we depart farther from the center in any direction. Furthermore, the major and minor axes of these

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Insert Figure 2 about here

---

ellipses would -- by virtue of the uncorrelatedness of ξ and $\hat{\theta}$ -- be parallel to the vertical (ξ) and horizontal ($\hat{\theta}$) reference axes of the rule space, respectively.

Recalling that for any given percentage ellipse, the lengths of the major and minor diameters are fixed multiples of the respective standard deviations

$$[\sum_{j=1}^{n} P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2]^{1/2} \text{ and } I(\hat{\theta})^{-1/2}$$

we may assert that the set of ellipses gives a complete characterization of the rule space. By this is meant that, once these ellipses are given, any response-pattern point can be classified as most likely being a random slip from one or another of the erroneous rules (or the correct one). We have only to determine, for a suitable percent value, which one of the several ellipses uniquely includes the given point.

Operational Classification Scheme

The geometrics scheme outlined above for classifying any given response-pattern point as being a "perturbation" from one or another of the rule points has a certain intuitive appeal (especially to those with high spatial ability!). However, it is obviously difficult if not infeasible to put it into practice. We, therefore, now describe the algebraic equivalent of the foregoing geometric clasification-decision rule, which is none other than the well-known minimum-D^2 rule, where D^2 is Mahalanobis' generalized squared-distance (Fukunaga, 1972; Tatsuoka, 1971). Then the

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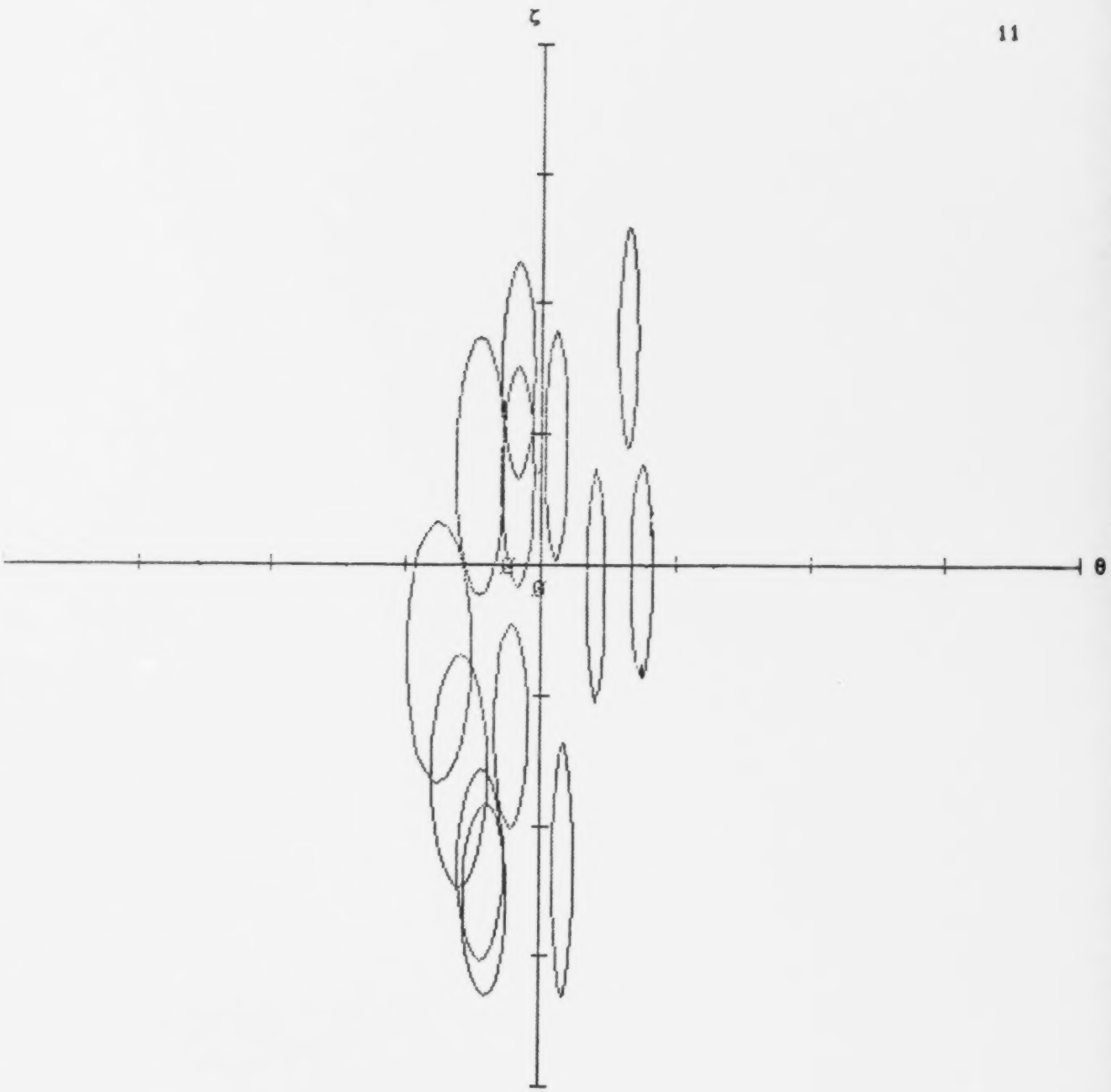

Figure 2: Fifteen Ellipses Representing Fifteen Error Types Randomly Chosen From Forty Sets of Ellipses

Bayes' decision rule for minimum error will be introduced.

Without loss of generality, we may suppose that a given response-pattern point $\underset{\sim}{x}$ has to be classified as representing a random slip from one of two rule points $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$. Let $\underset{\sim}{Y}$ be a point in the rule space corresponding to $\underset{\sim}{x}$, $\underset{\sim}{X} = \begin{bmatrix} \hat{\theta}_x \\ f(\underset{\sim}{x}) \end{bmatrix}$. The Mahalanobis distance of $\underset{\sim}{X}$ from each of the two rule points is

$$(15) \quad D^2_{xj} = [\, \underset{\sim}{X} - \underset{\sim}{R}_j \,]' \, \underset{\sim}{\Sigma}^{-1} \, [\, \underset{\sim}{X} - \underset{\sim}{R}_j \,] \quad (j=1,2)$$

where $\underset{\sim}{R}_1 = \begin{bmatrix} \hat{\theta}_{R_1} \\ f(\underset{\sim}{R}_1) \end{bmatrix}$ and $\underset{\sim}{R}_2 = \begin{bmatrix} \hat{\theta}_{R_1} \\ f(\underset{\sim}{R}_2) \end{bmatrix}$, and the variance-covariance matrix will be,

$$\underset{\sim}{\Sigma} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & \mathrm{var}(f(\underset{\sim}{x})) \end{bmatrix}$$

The decision rule is, of course, to classify $\underset{\sim}{x}$ as a perturbation from $\underset{\sim}{R}_1$ if $D^2_{x1} < D^2_{x2}$ and otherwise as a perturbation from $\underset{\sim}{R}_2$. However, the decision based on the Mahalanobis distances, $D^2_{x_1}$ and $D^2_{x_2}$ does not provide error probabilities of misclassificaton. The next section will discuss them.

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The Bayes Decision Rule for Minimum Error

Suppose R_1 and R_2 are two clusters of points corresponding to Rules 1 and 2, respectively. Let $\underset{\sim}{Y}$ be a vector $(\hat{\theta}, \zeta)$ corresponding to an observed response pattern $\underset{\sim}{x}$, and ζ be the standardized value of $f(\underset{\sim}{x})$, IRT-based caution index. Then the variance-covariance matrix Σ will be

$$\underset{\sim}{\Sigma} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & 1 \end{bmatrix}$$

A decision rule based on probabilities may be summarized as follows:

(16) If $\text{Prob}(R_1 \mid \underset{\sim}{Y}) > \text{Prob}(R_2 \mid \underset{\sim}{Y})$ then $\underset{\sim}{Y} \in R_1$ and

if $\text{Prob}(R_1 \mid \underset{\sim}{Y}) < \text{Prob}(R_2 \mid \underset{\sim}{Y})$ then $\underset{\sim}{Y} \in R_2$.

These posterior probabilities can be obtained from prior probabilities, $\text{Prob}(R_1)$ and $\text{Prob}(R_2)$, and the conditional density function $p(\underset{\sim}{Y} \mid R_i)$, $i=1,2$ as follows:

$$(17) \qquad \text{Prob}(R_i \mid \underset{\sim}{Y}) = \frac{p(\underset{\sim}{Y} \mid R_i)\ \text{Prob}(R_i)}{p(\underset{\sim}{Y})}$$

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Therefore, the decision rule can be expressed as follows:

(18) If $p(\underset{\sim}{Y} \mid R_1)\text{Prob}(R_1) > p(\underset{\sim}{Y} \mid R_2)\text{Prob}(R_2)$ then $\underset{\sim}{Y} \in R_1$

Otherwise, $\underset{\sim}{Y} \in R_2$.

This rule will be rewritten by using the likelihood ratio $\ell(\underset{\sim}{Y})$,

(19) If $$\ell(Y) = \frac{p(\underset{\sim}{Y} \mid R_1)}{p(\underset{\sim}{Y} \mid R_2)} > \frac{\text{Prob}(R_2)}{\text{Prob}(R_1)}$$, then $\underset{\sim}{Y} \in R_1$.

Otherwise, $\underset{\sim}{Y} \in R_2$.

Sometimes, it is convenient to take the negative log of the likelihood ratio in Expression (19), and rewrite it as Expression (20).

(20) If $h(\underset{\sim}{Y}) = -\ln \ell(\underset{\sim}{Y}) = -\ln(p(\underset{\sim}{Y} \mid R_1)) + \ln(p(\underset{\sim}{Y} \mid R_2))$

$< \ln [\ \text{Prob}(R_1) / \text{Prob}(R_2)\ ]$ then $\underset{\sim}{Y}$ belongs to R_1 .

However, the decision rule (20) does not lead to a perfect classification. As Overall (1972) states (p. 330)

> "Statistical classification decisions, like clinical diagnostic decisions, are only probabilistically correct. The clinician realizes this when he lists a secondary diagnosis. The statistician recognizes it more explicitly when he is able to assign a probability estimate to each classfication alternative."

The probability of error is the probability of $\underset{\sim}{Y}$ to be assigned to the wrong group, R_i.

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Let us denote the Posterior density function by $P(R_i \mid \underset{\sim}{Y})$, prior density function of R_i by $P(R_i)$ and let Γ_1 and Γ_2 be the regions such that if $\underset{\sim}{Y} \in \Gamma_1$ then $P(R_1 \mid \underset{\sim}{Y}) > P(R_2 \mid \underset{\sim}{Y})$ and
if $\underset{\sim}{Y} \in \Gamma_2$ then $P(R_1 \mid \underset{\sim}{Y}) < P(R_2 \mid \underset{\sim}{Y})$.

The probability of error is given by the following equation:

$$(21)\quad \varepsilon = \text{Prob}(\underset{\sim}{Y} \in \Gamma_2 \mid R_1)\, P(R_1) + \text{Prob}(\underset{\sim}{Y} \in \Gamma_1 \mid R_2)\, P(R_2) .$$

Let us denote the probability of $\underset{\sim}{Y}$ belonging to Γ_2 when $\underset{\sim}{Y}$ is from R_1 by ε_1, then

$$\varepsilon_1 = \text{Prob}(\underset{\sim}{Y} \in \Gamma_2 \mid R_1) = \int_{\Gamma_2} p(\underset{\sim}{Y} \mid R_1)\, d\underset{\sim}{Y},$$

Similarly, the probability of $\underset{\sim}{Y}$ belonging to Γ_1 when $\underset{\sim}{Y}$ is from R_2, ε_2 will be

$$\varepsilon_2 = \text{Prob}(Y \in \Gamma_1 \mid R_2) = \int_{\Gamma_1} p(\underset{\sim}{Y} \mid R_2)\, d\underset{\sim}{Y} .$$

Then expression (21) can be rewritten by $\varepsilon = \varepsilon_1 P(R_1) + \varepsilon_2 P(R_2)$, or more precisely

$$(22)\quad \varepsilon = P(R_1) \int_{\Gamma_2} p(\underset{\sim}{Y} \mid R_1)\, d\underset{\sim}{Y} + P(R_2) \int_{\Gamma_1} p(\underset{\sim}{Y} \mid R_2)\, d\underset{\sim}{Y} .$$

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That is, the total probability of errors is a weighted sum of the misclassification of samples from R_1 and R_2 into R_2 and R_1, respectively.

The integration of the conditional density function is necessary to get the error probability ε. The dimensionality of the conditional density function is often more than one, while the density function $p(\ell \mid R_i)$ of the likelihood ratio is one dimensional, so it is sometimes convenient to integrate the latter (Fukunaga, 1972). Hence, Equations (23) and (24) are used to obtain the error probabilities, ε_1 and ε_2:

$$\varepsilon_1 = \int_0^{P(R_2)/P(R_1)} p(\ell|R_1)\,d\ell \tag{23}$$

$$\varepsilon_2 = \int_{P(R_2)/P(R_1)}^{\infty} p(\ell|R_2)\,d\ell \tag{24}$$

If the density function $p(\underset{\sim}{Y} \mid R_i)$ is normal with expectations $\underset{\sim}{M}_i$ and covariance matrices $\underset{\sim}{\Sigma}_i$, the decision rule is summarized by the following statements:

(25) If $h(\underset{\sim}{Y}) = -\ln \ell(\underset{\sim}{Y})$

$$= \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_1)' \underset{\sim}{\Sigma}_1^{-1} (\underset{\sim}{Y} - \underset{\sim}{M}_1) - \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_2)' \underset{\sim}{\Sigma}_2^{-1}(\underset{\sim}{Y} - \underset{\sim}{M}_2)$$

$$+ \tfrac{1}{2} \ln \frac{|\Sigma_1|}{|\Sigma_2|} \begin{matrix} < \\ > \end{matrix} \ln \frac{P(R_1)}{P(R_2)} + Y \quad \begin{matrix} \in R_1 \\ \in R_2 \end{matrix} .$$

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If $\underset{\sim}{\Sigma}_1 = \underset{\sim}{\Sigma}_2 = \underset{\sim}{\Sigma}$, then $h(\underset{\sim}{Y})$ becomes a linear function of $\underset{\sim}{Y}$ and the decision rule has the following form if $\underset{\sim}{Y}$ follows a normal distribution:

$$(26)\quad h(\underset{\sim}{Y}) = \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_1)' \underset{\sim}{\Sigma}^{-1} (\underset{\sim}{Y} - \underset{\sim}{M}_1) - \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_2)' \underset{\sim}{\Sigma}^{-1} (\underset{\sim}{Y} - \underset{\sim}{M}_2)$$

$$= \tfrac{1}{2}((\underset{\sim}{M}_2' - \underset{\sim}{M}_1')\underset{\sim}{\Sigma}^{-1}\underset{\sim}{Y} - \underset{\sim}{Y}'\underset{\sim}{\Sigma}^{-1}(\underset{\sim}{M}_1 - \underset{\sim}{M}_2) + \underset{\sim}{M}_1'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}_1 - \underset{\sim}{M}_2'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}$$

$$= (\underset{\sim}{M}_2' - \underset{\sim}{M}_1')\underset{\sim}{\Sigma}^{-1}\underset{\sim}{Y} + \tfrac{1}{2}(\underset{\sim}{M}_1'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}_1 - \underset{\sim}{M}_2'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}_2) \gtrless \ln[P(R_1)/P(R_2)] = t.$$

then, $\quad Y \in \begin{cases} R_1 \\ \\ R_2 \end{cases}$

The error probability ε_1 is given by,

$$(27)\quad \varepsilon_1 = \int_t^{\infty} p(h(Y) \mid R_1)\,dh(\underset{\sim}{Y}) = \int_{\frac{t+\eta}{\sigma}}^{\infty} 1/\sqrt{2\pi}\ \exp\left(-\frac{z^2}{2}\right) dz$$

$$= 1 - \Psi\left(\frac{t+\eta}{\sigma}\right) .$$

where $t = \ln [p(R_1) / p(R_2)]$ and $\Psi(.)$ is the unit normal distribution. The conditional expectation of the likelihood function $h(\underset{\sim}{Y})$ is given by (28) and (29),

$$(28)\quad E(h(\underset{\sim}{Y}) \mid R_1) = -\tfrac{1}{2}(\underset{\sim}{M}_2 - \underset{\sim}{M}_1)' \underset{\sim}{\Sigma}^{-1} (\underset{\sim}{M}_2 - \underset{\sim}{M}_1) = -\eta$$

$$(29)\quad E(h(\underset{\sim}{Y}) \mid R_2 = +\tfrac{1}{2}(\underset{\sim}{M}_2 - \underset{\sim}{M}_1)' \underset{\sim}{\Sigma}^{-1} (\underset{\sim}{M}_2 - \underset{\sim}{M}_1) = +\eta$$

and, the variance of h(Y) is given by Equation (30):

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$$(30)\quad \sigma_i^2 = E[(h(Y) - \eta_i)^2 \mid R_i]$$

$$= (\underset{\sim}{M}_2 - \underset{\sim}{M}_1)'\Sigma^{-1}(M_2 - M_1) = 2\eta \quad .$$

Similarly, ε_2 can be obtained by calculating $1 - \Psi(\frac{\eta - t}{\sigma})$, i.e.,

$$(31)\quad \varepsilon_2 = \int_{-\infty}^{t} p(h(Y) \mid R_2)dh(\underset{\sim}{Y}) = 1 - \Psi(\frac{\eta - t}{\sigma}) \quad .$$

Illustration of the model with an example

A 40-item fraction subtraction test was given to 535 students at a local junior high school. A computer program adopting a deterministic strategy for diagnosing erroneous rules of operation in subtracting two fractions was developed on the PLATO system. The students' performances on the test were analyzed by the error-diagnostic program and summarized by Tatsuoka (1984a). In order to illustrate the rule space model and the decision rule described in the previous section, two very common erroneous rules (Tatsuoka, 1984a) are chosen to explain the model.

Rule 8. This rule is applicable to any fraction or mixed number. A student subtracts the smaller from the larger number in unequal corresponding parts and keeps corresponding equal parts as is in the answer. Examples are,

1. 4 4/12 - 2 7/12 = 2 3/12 = 2 1/4
2. 7 3/5 - 4/5 = 7 1/5
3. 3/4 - 3/8 = 3/4

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Rule 30. This rule is applicable to the subtraction of mixed numbers where the first numerator is smaller than the second numerator. A student reduces the whole-number part of the minuend by one and adds one to the tens digit of the numerator.

1. 4 4/12 - 2 7/12 = 3 14/12 - 2 7/12 = 1 7/12
2. 3 3/8 - 2 5/6 = 2 13/8 - 2 5/6 = 19/24
3. 7 3/5 - 4/5 = 6 13/5 - 4/5 = 2 9/5

These two rules are applied to 40 items and two sets of responses are scored by "right or wrong" scoring procedure. The binary score pattern made by Rule 8 is denoted by $\underset{\sim}{R}_8$ and the other made by Rule 30 is denoted by $\underset{\sim}{R}_{30}$.

Besides the two rule mentioned above, 38 different error types are identified by a task analysis. However, these error types do not necessarily represent microlevels of cognitive processes such as erroneous rules of operaton. They are somehow, defined more coarsely, like borrowing errors are grouped as a single error type, or the combination of borrowing and getting the least common multiple of two denominators is counted as one error type. In other words, 38 binary reponse patterns representing 38 error types are obtained.

The 535 students' responses on the 40 items are scored and used for estimating item parameters a_j and b_j by the maximum likelihood procedure. By using these a- and b-values, θ-values associated with the two rules and 38 error types are computed. Then corresponding ξ-values are calculated. Thus, 40 points, $(\hat{\theta}_k, \xi_k)$, k=1,...,40 are plotted in the rule space (Rule 8 is renumbered to 39 and Rule 30 to 40. It is only coincidence that the number of rules equals the number.

Insert Table 1 about here

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Table 1

The 40 Centroids Representing 40 different error types in Fraction Subtraction Tests (N = 535, n = 40)

Group	θ	ζ	No. of Items	Group	θ	ζ	No. of Items
1	-2.69	-.80	1	21	.24	-.89	22
2	-1.22	-.69	4	22	-.22	-1.23	14
3	-.75	-.68	8	23	.62	-1.55	32
4	-.46	.75	10	24	1.04	-.61	38
5	.11	.91	18	25	.75	-.05	34
6	.64	1.74	30	26	-.51	-1.62	10
7	-.17	1.48	13	27	-.87	-.56	6
8	.40	-.16	25	28	-1.99	1.01	2
9	.60	-.43	31	29	-.19	1.53	12
10	.57	-.24	29	30	-.24	2.74	10
11	.99	.72	37	31	-1.18	1.46	4
12	1.19	.86	39	32	-1.45	.58	4
13	-.60	-1.58	10	33	.64	1.74	30
14	-.44	-2.31	12	34	.57	-.66	31
15	-.18	.67	14	35	.59	-1.39	30
16	-.08	-1.81	16	36	-1.66	-1.96	4
17	.16	-.86	20	37	-.52	-.94	10
18	-.01	-2.12	18	38	-.32	-1.26	14
19	.09	-2.26	20	39	-.41	-2.57	13
20	.29	-1.51	24	40	.17	-2.34	22

*These items will have the score of 1, otherwise the score will be 0.

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Now, two students A and B who used Rules 8 and 30 for a subset of 40 items are selected. This was possible because their performances are diagnosed independently by the error-diagnostic system BPFBUG mentioned in Tatsuoka (1984b). The circles shown in Figure 3 represent A and B. Their Mahalanobis distances, D^2, to the 40 centroids are calculated respectively and the smallest values of two distances, D^2 , are selected to compute probabilities of errors. Table 2 summarizes the results.

Insert Table 2 & Figure 3 about here

The D^2 values of Student A to Sets 40 and 19 are 0.008 and 0.119, respectively, and both the values are small enough to judge that A may be classified to either of the sets. Since D^2 follows the χ^2-distribution with two degrees of freedom (Tatsuoka, 1971), the null hypotheses that $D^2_{(A,Set\ 40)} = 0$ and $D^2_{(A,Set\ 19)} = 0$ cannot be rejected at, say $\alpha = .25$. The error probabilities ε_1 and ε_2 are .581, .266, respectively. Therefore, we conclude A belongs to Set 19 although $D^2_{(A,Set\ 40)}$ is smaller than $D^2_{(A,Set\ 19)}$. This happened because the prior probability of Prob(Set 40) is smaller than that of Prob(Set 19), where the threshold value, t, is determined as follows:

$$t = -\ell n \left[\text{Prob(Set 40)} \,/\, \text{Prob(Set 19)} \right]$$

and $$\text{Prob(Set k)} \propto (1/2\pi) \exp\left[-(\hat{\theta}_k, \zeta_k)' \, \Sigma_k^{-1} \, (\hat{\theta}_k, \zeta_k)/2 \right] .$$

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Table 2

Summary of Classification Results of Students A and B

	Student A		Student B	
D^2	$D^2_{A,\ Set\ 40}$	.008	$D^2_{B,\ Set\ 39}$	.021
	$D^2_{A,\ Set\ 19}$	.119	$D^2_{B,\ Set\ 14}$	.135
ε_1	.581		.979	
ε_2	.266		.010	
η	.088		.040	
t	-.174		-.613	

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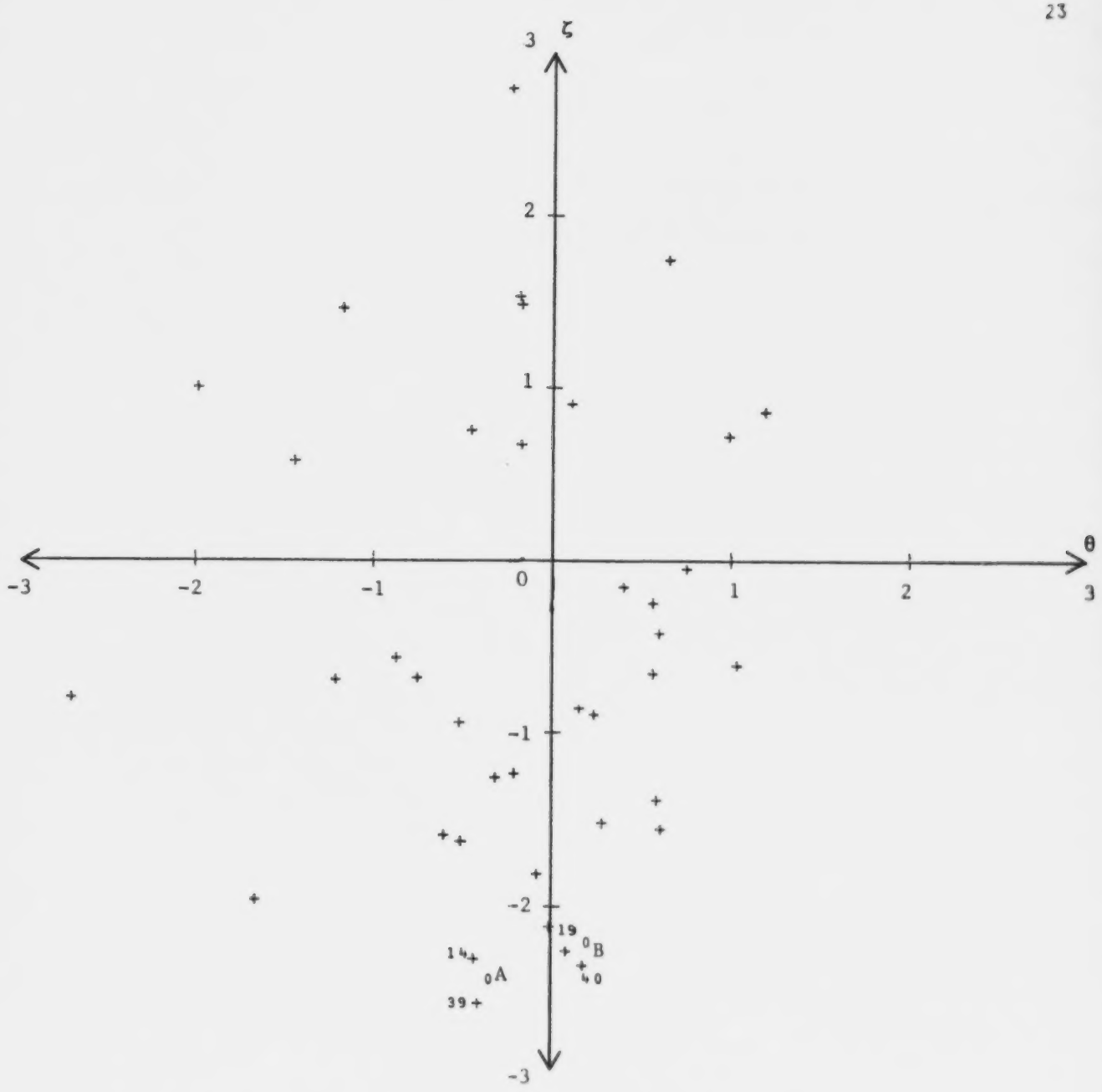

Figure 3: Forty Centroids (+) Representing Different Error Types Determined by a Detailed Task Analysis and Students A and B ($_0$).

Discussion

A new probabilistic model that is capable of measuring cognitive-skill acquisition, and of diagnosing erroneous rules of operation in a procedural domain was introduced by Tatsuoka and her associates (Tatsuoka, 1985; Tatsuoka & Baillie, 1982; Tatsuoka & Tatsuoka, 1982; Tatsuoka, 1983; Tatsuoka, 1984a). The model, called rule space, involves two important components: 1) determination of a set of bug distributions, or in other words, bug density functions representing clusters around the rules, and 2) establishment of decision rules for classifying an observed response pattern into one of the clusters around the rules and computing error probabilities. If each cluster around a rule can be described by a bivariate normal distribution of θ and ζ, then application of the techniques available in the theory of statistical classification and pattern recognition is fairly straightforward and easy.

This study introduces the fact that the cluster around the rule consisting of the response patterns resulting from one, two,..., several slips away from perfect applicaton of the rule indeed follows a compound binomial distribution with centroid (θ_R, ζ_R) and variance $\sum_{j=1}^{n} p_j q_j$, where p_j (j=1,...,n is the probability of having a slip from Rule R for item j. The values of p_j and q_j are approximated by the logistic probabilities $P_j(\theta_R)$ and $Q_j(\theta_R)$, j=1,...,n, in this study instead of estimating them from the dataset. Plausibility of the approximation of the slip probabilities associated with each erroneous rule by the logistic function is left as a future topic of investigation, although the fit with data seems to be good.

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The determination of a set of ellipses representing clusters around the rules can be automatic after all the erroneous rules are discovered. Many researchers in cognitive science and artificial intelligence have started constructing error diagnostic systems in various domains in this decade. Expert teachers usually know their students' errors, as well as the weaknesses and strengths of each child's knowledge structure. Since the model does not require a large-scale computation such as strategies commonly used in the area of artifical intelligence do, the rule-space model is helpful in more general areas of research and teaching, and for those who have microcomputers for testing their hypotheses, validating their data with probabilistically-sound information, and evaluating their teaching methods and materials. Moreover, the model can be "intelligent" in the sense that the researcher can improve and modify the information for the cluster ellipses as they get more new students whose performances they can study.

The set of ellipses can represent many things besides erroneous rules. They can represent specific contents of some domain, usage errors in the language arts, or processes required in algebra. However, further research is necessary to develop methods for determining the set of ellipses other than relying on an expert teacher. The method must be efficient and compatible with the recent theories of human cognition and learning.

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Personnel Analysis Division
AF/MPXA
5C360, The Pentagon
Washington, DC 20330

Air Force Human Resources Lab
AFHRL/MPD
Brooks AFB, TX 78235

Air Force Office
of Scientific Research
Life Sciences Directorate
Bolling Air Force Base
Washington, DC 20332

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
NAVTRAEQUIPCEN
Orlando, FL 32813

Dr. Ed Aiken
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. John R. Anderson
Department of Psychology
Carnegie-Mellon University
Pittsburgh, PA 15213

Technical Director
Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
Israel

Dr. Werner Birke
Personalstammamt
der Bundeswehr
D-5000 Koeln 90
WEST GERMANY

Code N711
Attn: Arthur S. Blaiwes
Naval Training Equipment Center
Orlando, FL 32813

Dr. R. Darrell Bock
University of Chicago
Department of Education
Chicago, IL 60637

Mr. Arnold Bohrer
Psychological Research Section
Caserne Petits Chateau
CRS
1000 Brussels
BELGIUM

Dr. Nick Bond
Office of Naval Research
Liaison Office, Far East
APO San Francisco, CA 96503

Dr. Robert Breaux
Code N-095R
NAVTRAEQUIPCEN
Orlando, FL 32813

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. John S. Brown
XEROX Palo Alto Research
Center
3333 Coyote Road
Palo Alto, CA 94304

Dr. Glenn Bryan
6208 Poe Road
Bethesda, MD 20817

CTB/McGraw-Hill Library
2500 Garden Road
Monterey, CA 93940

CDR Mike Curran
Office of Naval Research
800 N. Quincy St.
Code 270
Arlington, VA 22217-5000

Mr. Timothy Davey
University of Illinois
Educational Psychology
Urbana, IL 61801

Dr. Dattprasad Divgi
Syracuse University
Department of Psychology
Syracuse, NY 13210

Dr. Hei-Ki Dong
Ball Foundation
800 Roosevelt Road
Building C, Suite 206
Glen Ellyn, IL 60137

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Patricia A. Butler
NIE Mail Stop 1806
1200 19th St., NW
Washington, DC 20208

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
NAVOP 01B7
Washington, DC 20370

Director
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Chief of Naval Education
and Training
Liason Office
AFHRL
Operations Training Division
Williams AFB, AZ 85224

Chief of Naval Education
and Training
Liason Office
Air Force Human Resource Laboratory
Operations Training Division
Williams AFB, AZ 85224

Assistant Chief of Staff
Research, Development,
Test, and Evaluation
Naval Education and
Training Command (N-5)
NAS Pensacola, FL 32508

Dr. Hans Crombag
University of Leyden
Education Research Center
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

Dr. John Ellis
Navy Personnel R&D Center
San Diego, CA 92252

Dr. Richard Elster
Deputy Assistant Secretary
of the Navy (Manpower)
Washington, DC 20350

Dr. Susan Embertson
University of Kansas
Psychology Department
Lawrence, KS 66045

ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Marshall J. Farr
2520 North Vernon Street
Arlington, VA 22207

Dr. Richard L. Ferguson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Dr. Dexter Fletcher
University of Oregon
Computer Science Department
Eugene, OR 97403

BEST COPY AVAILABLE

Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

Dr. Bob Frey
Commandant (G-P-1/2)
USCG HQ
Washington, DC 20593

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01002

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenshaftliches
Universitatsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152

Prof. Lutz F. Hornke
Universitat Dusseldorf
Erziehungswissenschaftliches
Universitatsstr. 1
Dusseldorf 1
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Dr. Earl Hunt
Department of Psychology
University of Washington
Seattle, WA 98105

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Zachary Jacobson
Bureau of Management Consulting
365 Laurier Avenue West
Ottawa, Ontario K1A 0S5
CANADA

Dr. Douglas H. Jones
Advanced Statistical
Technologies Corporation
10 Trafalgar Court
Lawrenceville, NJ 08148

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. Norman J. Kerr
Chief of Naval Education
and Training
Code 00A2
Naval Air Station
Pensacola, FL 32508

Dr. William Koch
University of Texas-Austin
Measurement and Evaluation
Center
Austin, TX 78703

Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Anita Lancaster
Accession Policy
OASD/MI&L/MP&FM/AP
Pentagon
Washington, DC 20301

Dr. Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Jerry Lehnus
OASD (M&RA)
Washington, DC 20301

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Alan M. Lesgold
Learning R&D Center
University of Pittsburgh
Pittsburgh, PA 15260

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat 23
9712GC Groningen
The NETHERLANDS

Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

Dr. Robert Lockman
Center for Naval Analysis
200 North Beauregard St.
Alexandria, VA 22311

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

Dr. William L. Maloy (02)
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Kneale Marshall
Operations Research Department
Naval Post Graduate School
Monterey, CA 93940

Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Dr. James McBride
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Clarence McCormick
HQ, MEPCOM
MEPCT-P
2500 Green Bay Road
North Chicago, IL 60064

Mr. Robert McKinley
University of Toledo
Departmentof Educational Psychology
Toledo, OH 43606

Dr. Barbara Means
Human Resources
Research Organization
1100 South Washington
Alexandria, VA 22314

Dr. Arthur Melmed
724 Brown
U. S. Department of Education
Washington, DC 20208

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr William Montague
NPRDC Code 13
San Diego, CA 92152

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152

Headquarters, Marine Corps
Code MPI-20
Washington, DC 20380

Spec. Asst. for Research, Experimental Programs, & Academic Programs
Naval Technical Training Command
(Code 016)
NAS Memphis (75)
Millington, TN 38054

Program Manager for Manpower,
Personnel, and Training
NAVMAT 0722
Arlington, VA 22217-5000

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Dr. William E. Nordbrock
FMC-ADCO Box 25
APO, NY 09710

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Director, Training Laboratory
NPRDC (Code 05)
San Diego, CA 92152

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NPRDC (Code 06)
San Diego, CA 92152

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Human Factors & Organizational Systems Lab.
NPRDC (Code 07)
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San Diego, CA 92152

Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harry F. O'Neil, Jr.
Training Research Lab
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Stellan Ohlsson
Learning R & D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 1521

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

Office of Naval Research
Code 442PT
800 N. Quincy Street
Arlington, VA 22217-5000
(5 Copies)

Special Assistant for Marine Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217-5000

Psychologist
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101

Commanding Officer
Army Research Institute
ATTN: PERI-BR (Dr. J. Orasanu)
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
AFHRL/MOAN
Brooks AFB, TX 78235

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Cirle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dr. James W. Pellegrino
University of California, Santa Barbara
Department of Psychology
Santa Barabara, CA 93106

Dr. Roger Pennell
Air Force Human Resources Laboratory
Lowry AFB, CO 80230

Military Assistant for Training and Personnel Technology
OUSD (R & E)
Room 3D129, The Pentagon
Washington, DC 20301

Administrative Sciences Department
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research
Naval Postgraduate School
Monterey, CA 93940

Dr. Joseph Psotka
ATTN: PERI-IC
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Fred Reif
Physics Department
University of California
Berkeley, CA 94720

Dr. Lauren Resnick
Learning R & D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 1521

Dr. Mary S. Riley
Program in Cognitive Science
Center for Human Information Processing
University of California
La Jolla, CA 92093

Dr. Bernard Rimland
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Mr. Robert Ross
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Lawrence Rudner
403 Elm Avenue
Takoma Park, MD 20012

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
Knoxville, TN 37916

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152

Dr. Robert Sasmor
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Walter Schneider
University of Illinois
Psychology Department
603 E. Daniel
Champaign, IL 61820

Lowell Schoer
Psychological & Quantitative Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Judah L. Schwartz
MIT
20C-120
Cambridge, MA 02139

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Sylvia A. S. Shafto
National Institute of Education
1200 19th Street
Mail Stop 1805
Washington, DC 20208

Dr. Joyce Shields
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Kazuo Shigemasu
7 9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. H. Wallace Sinaiko
Manpower Research and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Alfred F. Smode
Senior Scientist
Code 7B
Naval Training Equipment Center
Orlando, FL 32813

Dr. Richard Snow
Liaison Scientist
Office of Naval Research Branch Office, London
Box 39
FPO New York, NY 09510

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Robert Sternberg
Department of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520

Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

BEST COPY AVAILABLE

Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

Maj. Bill Strickland
AF/MPXOA
4E168 Pentagon
Washington, DC 20330

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152

Dr. John Tangney
AFOSR/NL
Bolling AFB, DC 20332

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Headquarters, U. S. Marine Corps
Code MPI-20
Washington, DC 20380

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 310
St. Paul, MN 55114

Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08540

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Mr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
HumRRO
300 North Washington
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

Dr. Donald Weitzman
MITRE
1820 Dolley Madison Blvd.
MacLean, VA 22102

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90007

German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraeftesmt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Ms. Marilyn Wingersky
Educational Testing Service
Princeton, NJ 08541

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152

Dr. Frank Withrow
U. S. Office of Education
400 Maryland Ave. SW
Washington, DC 20202

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Joe Yasatuke
AFHRL/LRT
Lowry AFB, CO 80230

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
Memory & Cognitive
Processes
National Science Foundation
Washington, DC 20550

Dr. Leigh Burstein
Department of Education
University of California
Los Angeles, CA 90024

Dr. Norman Cliff
Department of Psychology
University of California
University Park
Los Angeles, CA 90007

Dr. Lee Cronbach
Department of Education
Stanford University
Stanford, CA 94305

Dr. James Greeno
LRDC
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15213

Dr. Anthony J. Nitko
School of Education
Educational Research
Methodology
University of Pittsburgh
5C03 Forbes Quadrangle
Pittsburgh, PA 15260

Dr. Ross Traub
Ontario Institute for
Studies in Education
252 Bloor St. W.
Toronto, Ontario
CANADA M5S 1V6

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